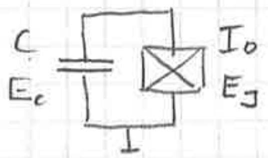
Transmon Qubit



$E_J \gg E_c$

problem: charge noise

two-level fluctuators

$$\mathcal{H} = \frac{Q^2}{2C} - \frac{I_0 \phi_0}{2\pi} \cos\left(2\pi \frac{\phi}{\phi_0}\right)$$

with $\{Q, \phi\} = i\hbar$

using $N := \frac{Q}{2e}$ $\quad \delta := 2\pi \frac{\phi}{\phi_0}$ $\quad \left(\phi_0 = \frac{h}{2e}\right)$

$$\mathcal{H} = 4E_c N^2 - E_J \cos\delta \qquad \text{with } \{N, \delta\} = i$$

expanding the cosine

$$\mathcal{H} = 4E_c N^2 - E_J \left(1 - \frac{\delta^2}{2!} + \frac{\delta^4}{4!} - \dots\right)$$

- the first term is constant and can be neglected
- up to the second term, the problem is a harmonic oscillator

$$\mathcal{H} = 4E_c N^2 + \frac{E_J}{2} \delta^2$$

with $N = \sqrt[4]{\frac{E_J}{8E_c}} \frac{1}{\sqrt{2}} (a + a^\dagger)$

$$\delta = \sqrt[4]{\frac{8E_c}{E_J}} \frac{i}{\sqrt{2}} (a - a^\dagger)$$

$$\mathcal{H} = \sqrt{8E_c E_J} \left(a^\dagger a + \frac{1}{2}\right)$$

treat the third term as a perturbation

$$\Delta E_m = - E_J \frac{1}{24} \langle m | \delta^4 | m \rangle$$

$$= - E_J \left(\frac{8E_c}{E_J} \right) \frac{1}{24} \frac{1}{4} \underbrace{\langle m | (a - a^\dagger)^4 | m \rangle}_{= 6m^2 + 6m + 3}$$

$$= - \frac{E_c}{4} \left(2m^2 + 2m + 1 \right)$$

$$\Delta E_m - \Delta E_{m-1} = - m E_c$$

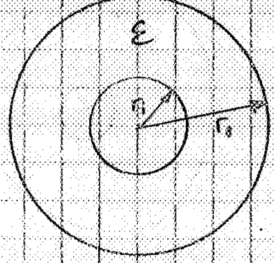
- small dependence on charge noise
  small E_c (large C)
  exponential

- large non-linearity, large E_c (small C)
  linear

trade off: $E_J / E_c \approx 80$

Circuit Cavity QED

Transmission line

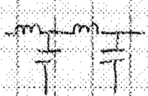


$$\vec{E}(\rho, \varphi, z) = \frac{V_0}{\ln(r_0/r_1)} \frac{\vec{e}_\rho}{\rho} e^{i\omega t - i\beta z}$$

$$\vec{H}(\rho, \varphi, z) = \frac{I_0}{2\pi} \frac{\vec{e}_\varphi}{\rho} e^{i\omega t - i\beta z}$$ TEM modes

$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$

$$Z_0 = \frac{V_0}{I_0} = \frac{\ln(r_0/r_1)}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \qquad \beta = \frac{\omega}{\frac{c}{\sqrt{\varepsilon\mu}}}$$ no dispersion



capacitance per unit length

$$C^* = \frac{1}{V_0^2} \int_\Sigma \varepsilon_r \varepsilon_0 \vec{E}\,\vec{E}^*\,dx\,dy = \frac{2\pi}{\ln(r_0/r_1)} \varepsilon_r \varepsilon_0$$

inductance per unit length

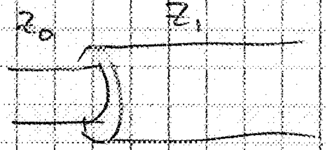
$$L^* = \frac{1}{I_0^2} \int_\Sigma \mu_r \mu_0 \vec{H}\,\vec{H}^*\,dx\,dy = \frac{\ln(r_0/r_1)}{2\pi} \mu_r \mu_0$$

$$Z_0 = \sqrt{\frac{L^*}{C^*}}$$

semi infinite transmission line is physically identical to a resistor of $R = Z_0$

LII, 2

impedance missmatch

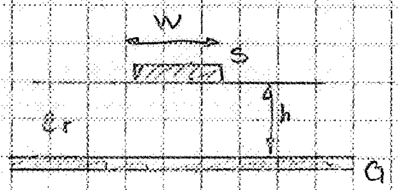


reflection coefficient

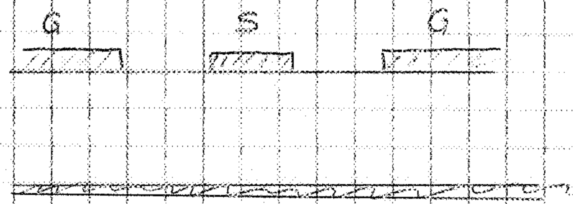
$$\Gamma = \frac{Z_1 - Z_0}{Z_1 - Z_0}$$

other transmission lines

microstrip



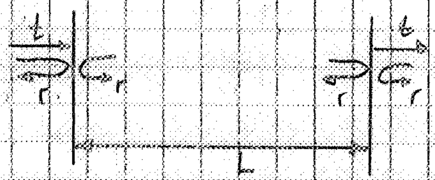
coplanar waveguide



transmission line resonator

slides

general resonance

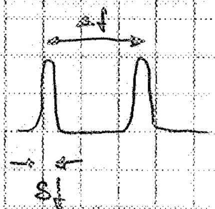


$$S = t\, e^{i\beta L}\, t + t\, e^{i\beta L}\, r\, e^{i\beta L}\, r\, e^{i\beta L}\, t + \ldots$$

$$= t^2 e^{i\beta L} \left(\sum_n \left(r\, e^{i\beta L} \right)^{2n} \right)$$

geometrical series

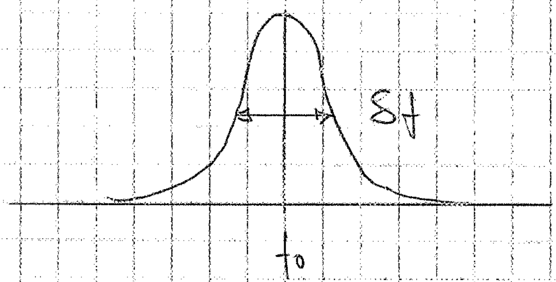
$$= a \frac{1}{1 - \left(r e^{i\beta L} \right)^2}$$



resonance if $\beta L \approx m\pi$

$$S_L = \frac{1}{f - f_0 - i\gamma}$$

$$|S_L| = \frac{1}{(f - f_0)^2 + \gamma^2}$$ Lorentz curve



δf full width halfmax

$\delta f = 2\gamma$ Quality factor $Q = \frac{f_0}{\delta f}$

Finesse $= \frac{\Delta f}{\delta f}$

L11.4

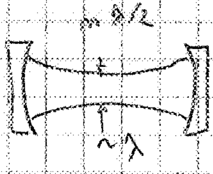
transmission line cavity

$\boxed{\text{slide}}$

$$Q_{cpl} = \frac{\pi}{4m} \frac{1}{\left(C_{cpl}\, 2\pi\, f_0\, Z_0\right)^2} \qquad f_0 = \frac{c/\sqrt{\varepsilon_r \mu_r}}{2L}$$

$$Q_{tot}^{-1} = Q_{cpl}^{-1} + Q_{int}^{-1}$$

optical cavity



fields for one photon

$$\frac{1}{2} h f_0 = \frac{\varepsilon_0}{2} \int \vec{E}^2\, dV \approx \frac{\varepsilon_0}{2} E_0 V$$

$$E_0 = \sqrt{\frac{h f_0}{V \varepsilon_0}}$$

cavity volume $\quad V = \pi r^2 \lambda/2 \qquad \lambda = \frac{c}{f_0}$

$$\Longrightarrow \quad E_0 = \frac{f_0}{r} \sqrt{\frac{h}{\frac{\pi}{2} \varepsilon_0 c}}$$

$$f_0 = 5\ \text{GHz} \qquad r = 10\ \mu\text{m}$$

$$h f_0 = 3.3 \cdot 10^{-24}\ \text{J}$$

$$E_0 = 0.2\ \text{V/m}$$

L11,5

comparing to conventional cavity

$V \propto \lambda^2$ $\quad$ E_0 is larger by factor $\frac{\lambda}{r}$

superconductivity $Q \sim 5 \quad 10^5$

slide

$5\ GHz \cdot h = k_B \cdot 240mK$